Lecture 4: An introduction to input-output analysis and its use in consumption-based accounting

Dr Anne Owen
a.owen@leeds.ac.uk
@dr_anneowen

Homework: Introduction to Matrix Algebra using Matlab

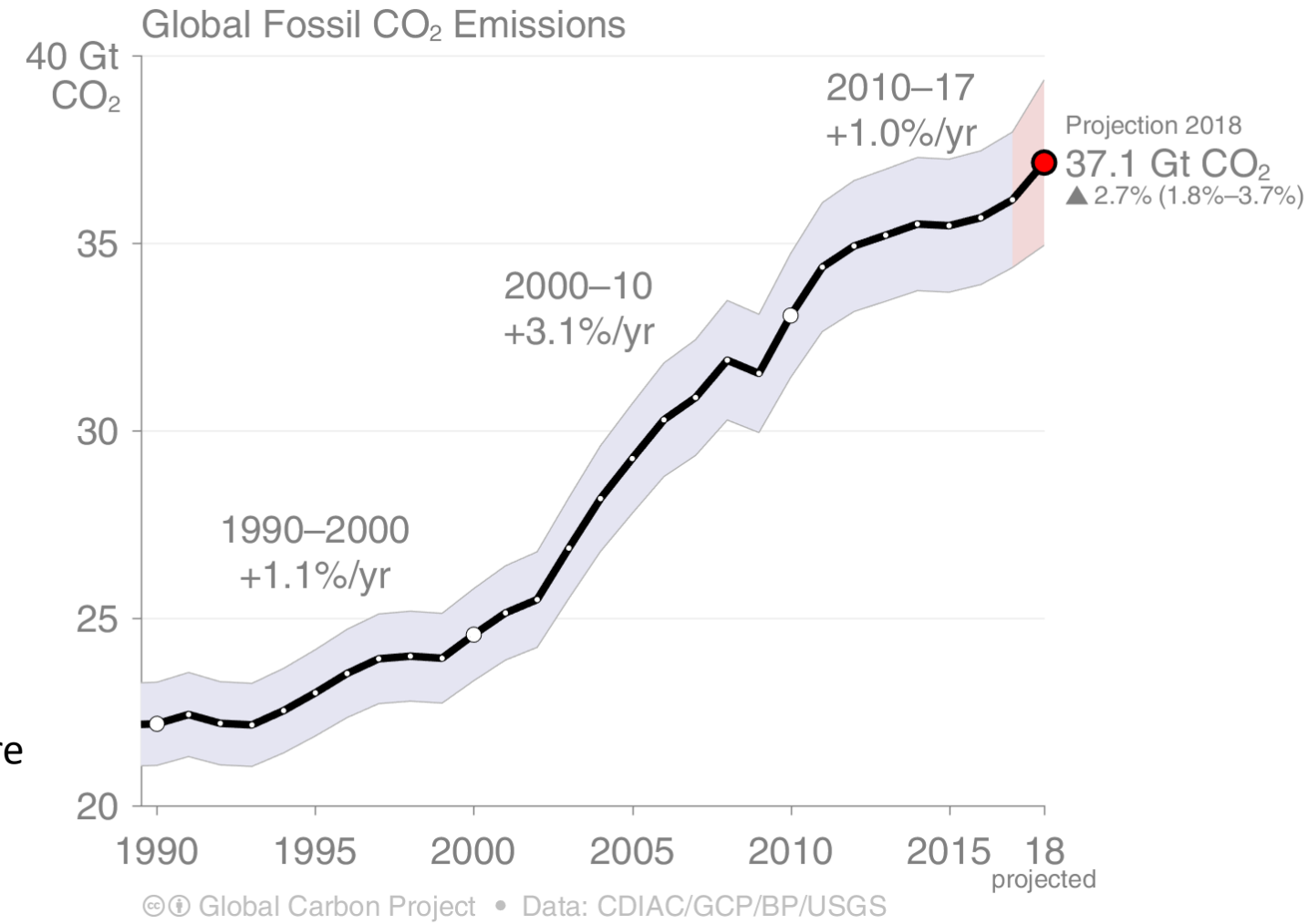
- Any problems?
- The essentials:
    - Adding and subtracting matrices
    - Matrix multiplication has two forms – product and dot product
    - What happens when you multiply a matrix by its inverse?

Lecture (45 mins)

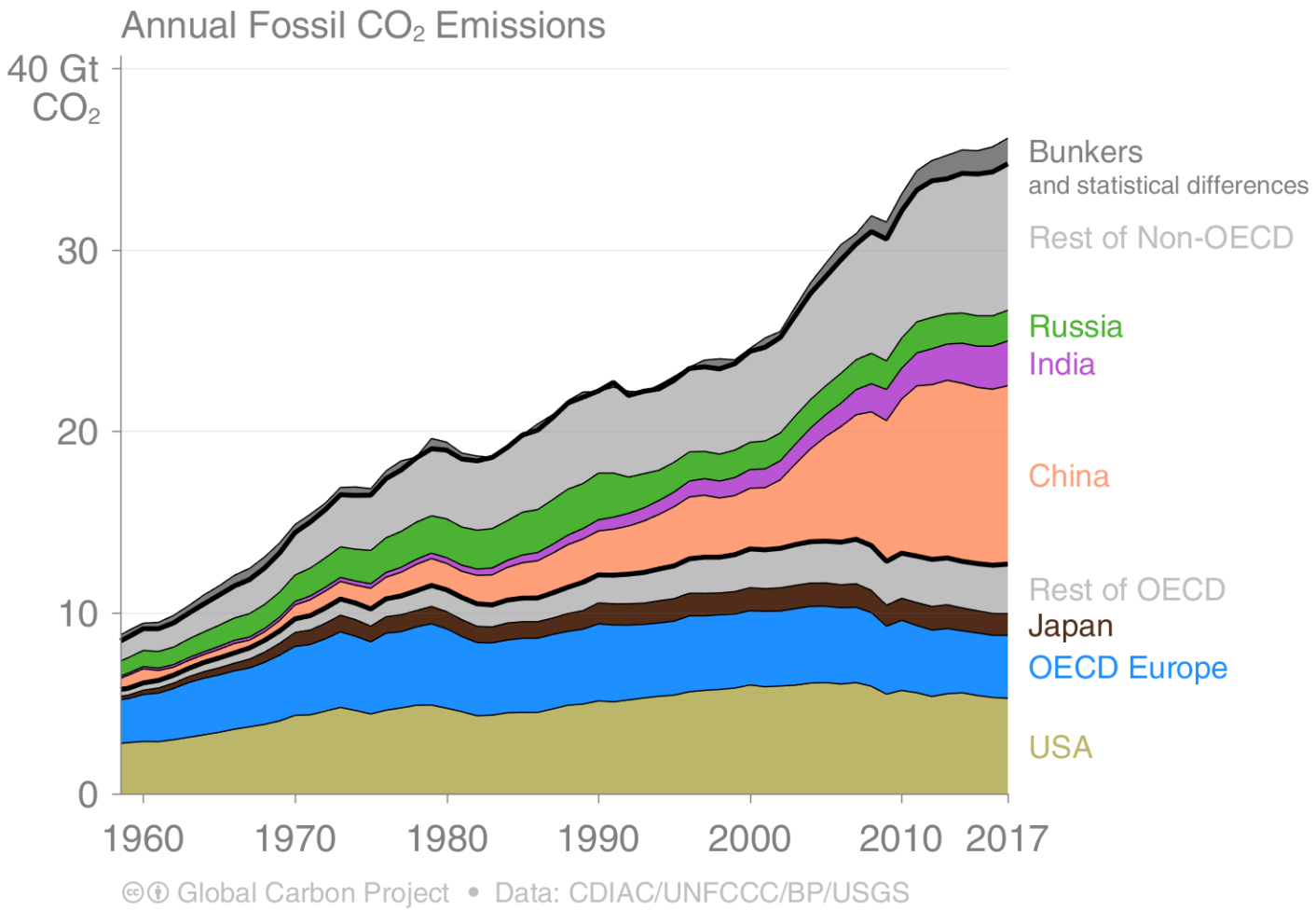
- What does a consumption-based approach to emissions accounting tell us?
- How can we account for emissions from consumption?
- Input-output analysis

Carbon emissions are increasing, but what are the drivers of this increase?

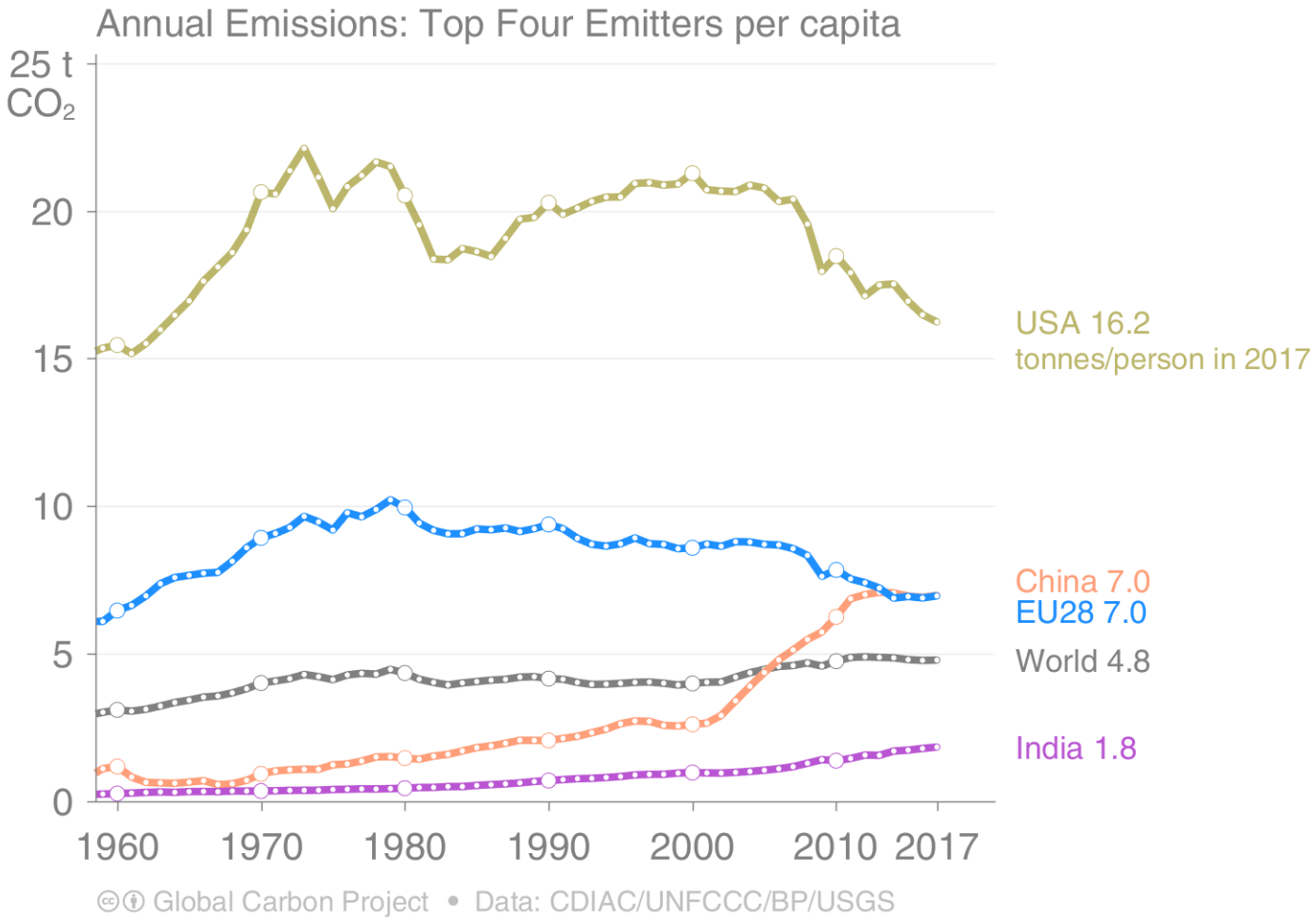
- Increase in industrial output?
- Population?
- Increase in air travel?
- Increase in consumption?
- Production becoming more inefficient?



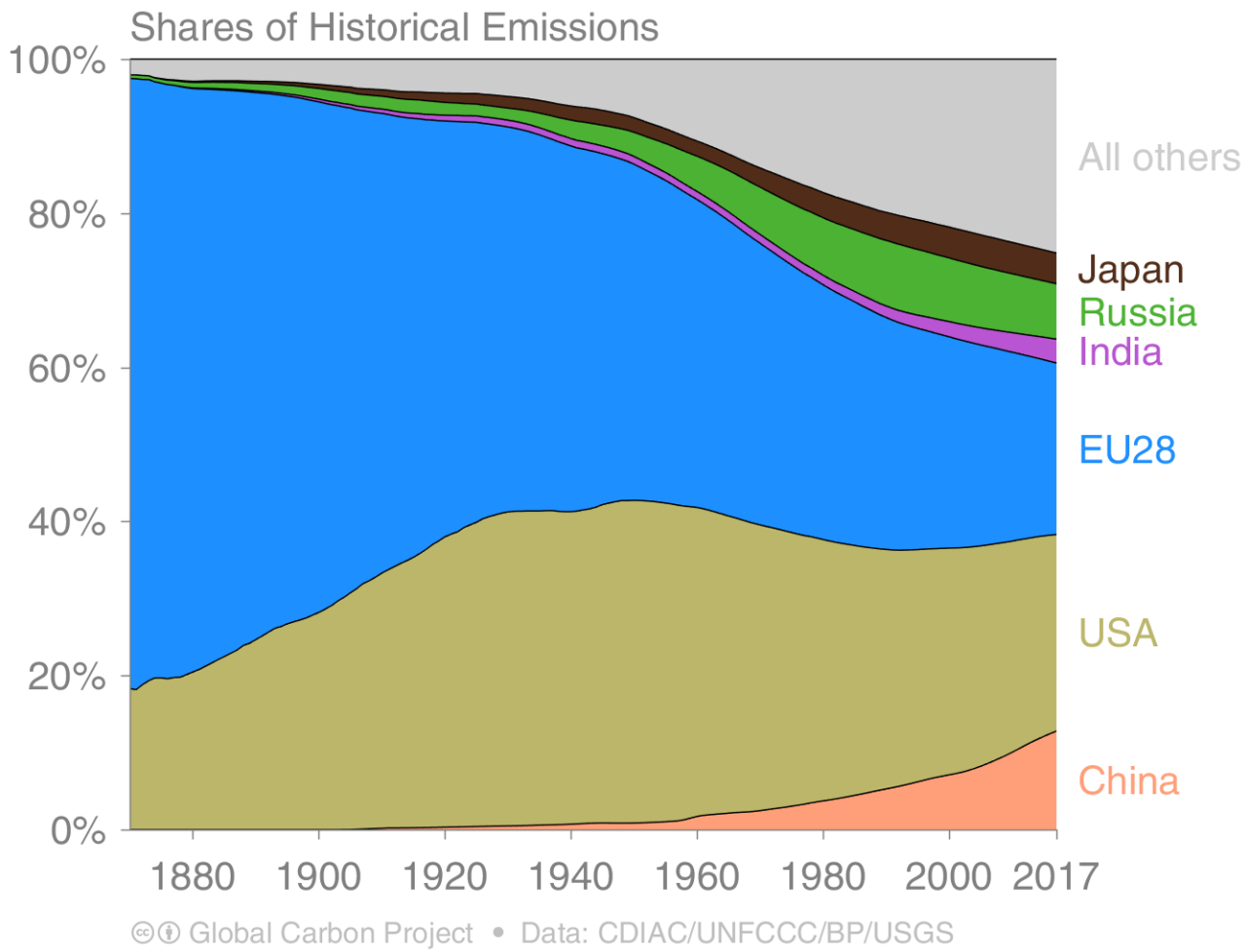
Emissions from OECD countries are about the same as in 1990
Emissions from non-OECD countries have increased rapidly in the last decade



Countries have a broad range of per capita emissions reflecting their national circumstances

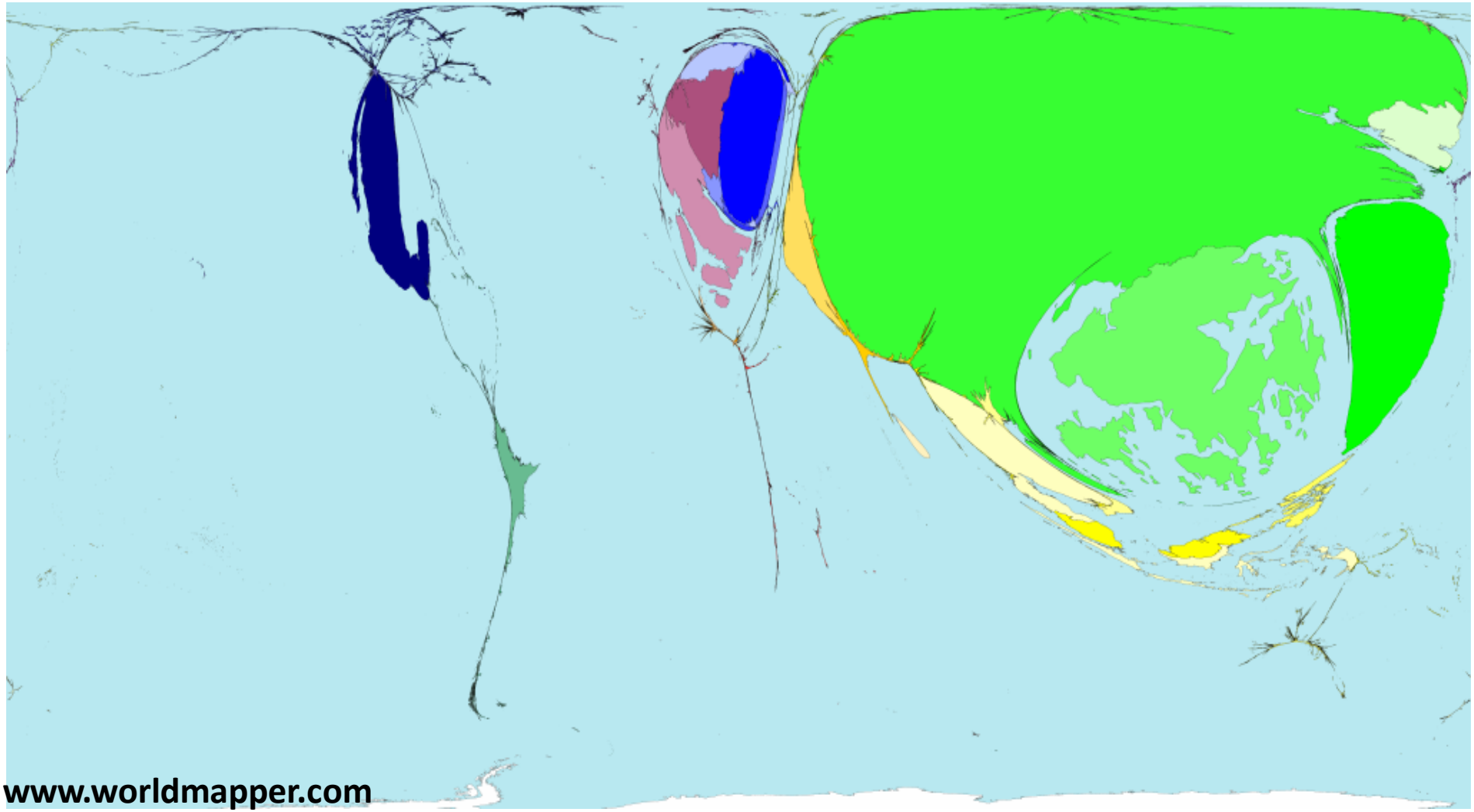


Cumulative emissions from fossil-fuel and industry were distributed (1870–2017): USA 26%, EU28 22%, China 13%, Russia 7%, Japan 4% and India 3%



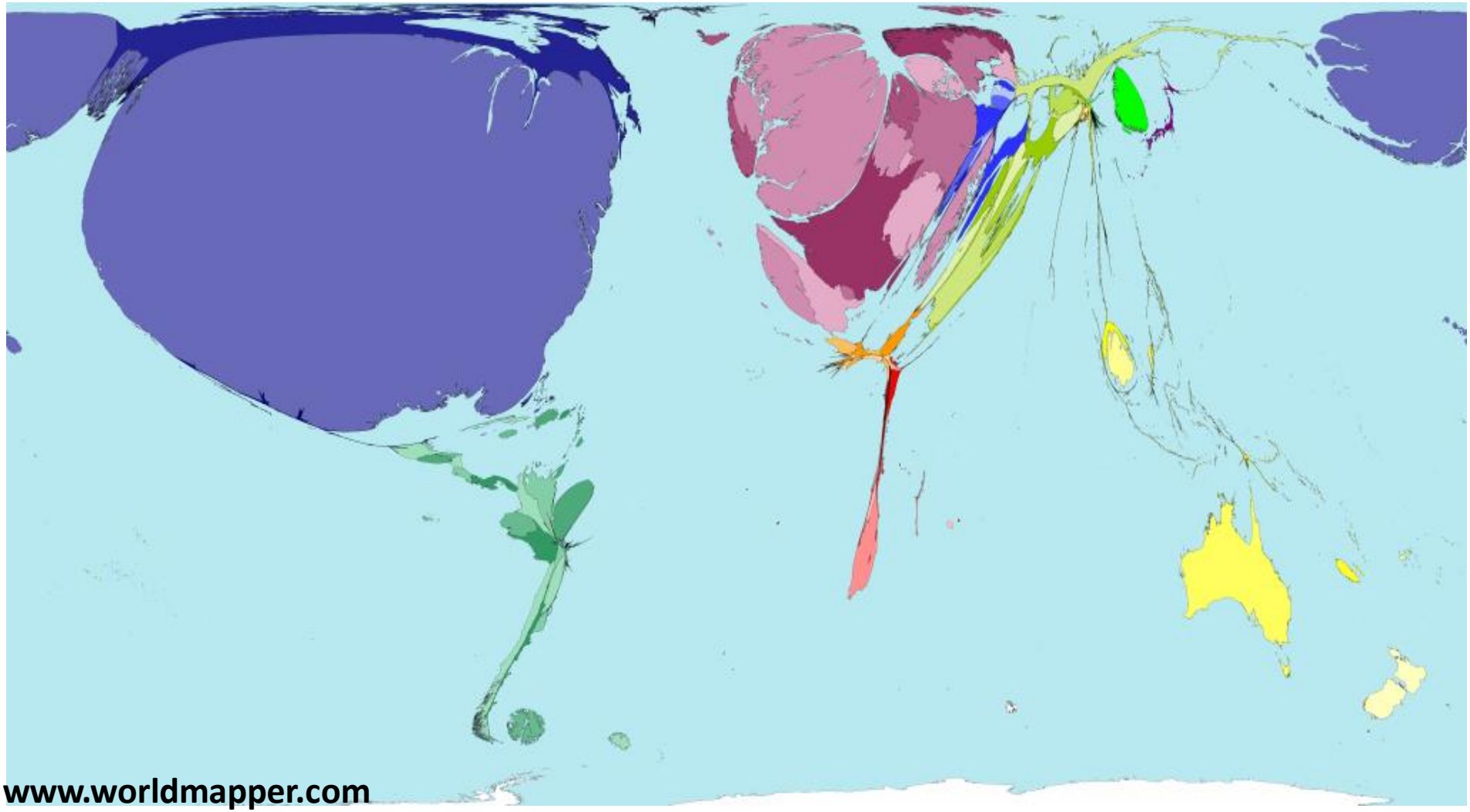
Historically, the EU and US still contribute the greatest share of emitted CO_2

Where are the world's toys made?



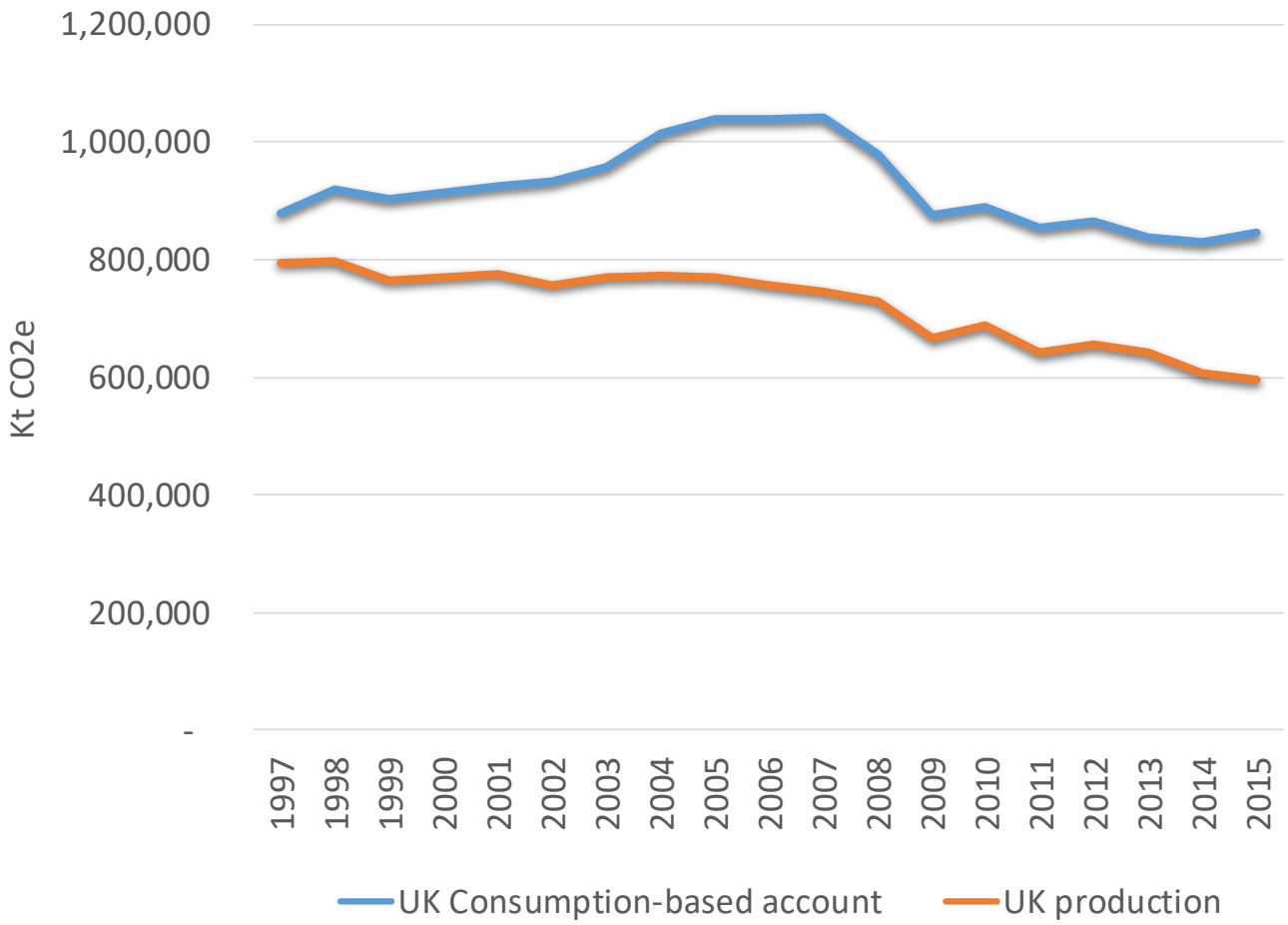
www.worldmapper.com

Who buys the toys?



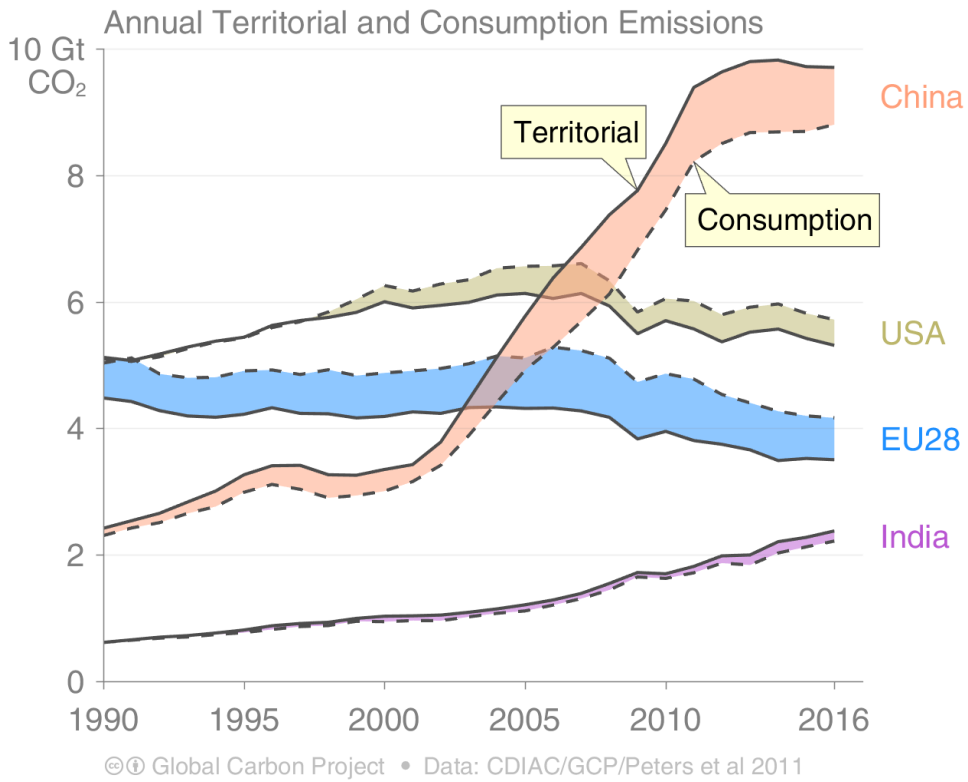
www.worldmapper.com

The UK


1,200,000
1,000,000
800,000
600,000
400,000
200,000
-
Kt CO2e
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
UK Consumption-based account
UK production

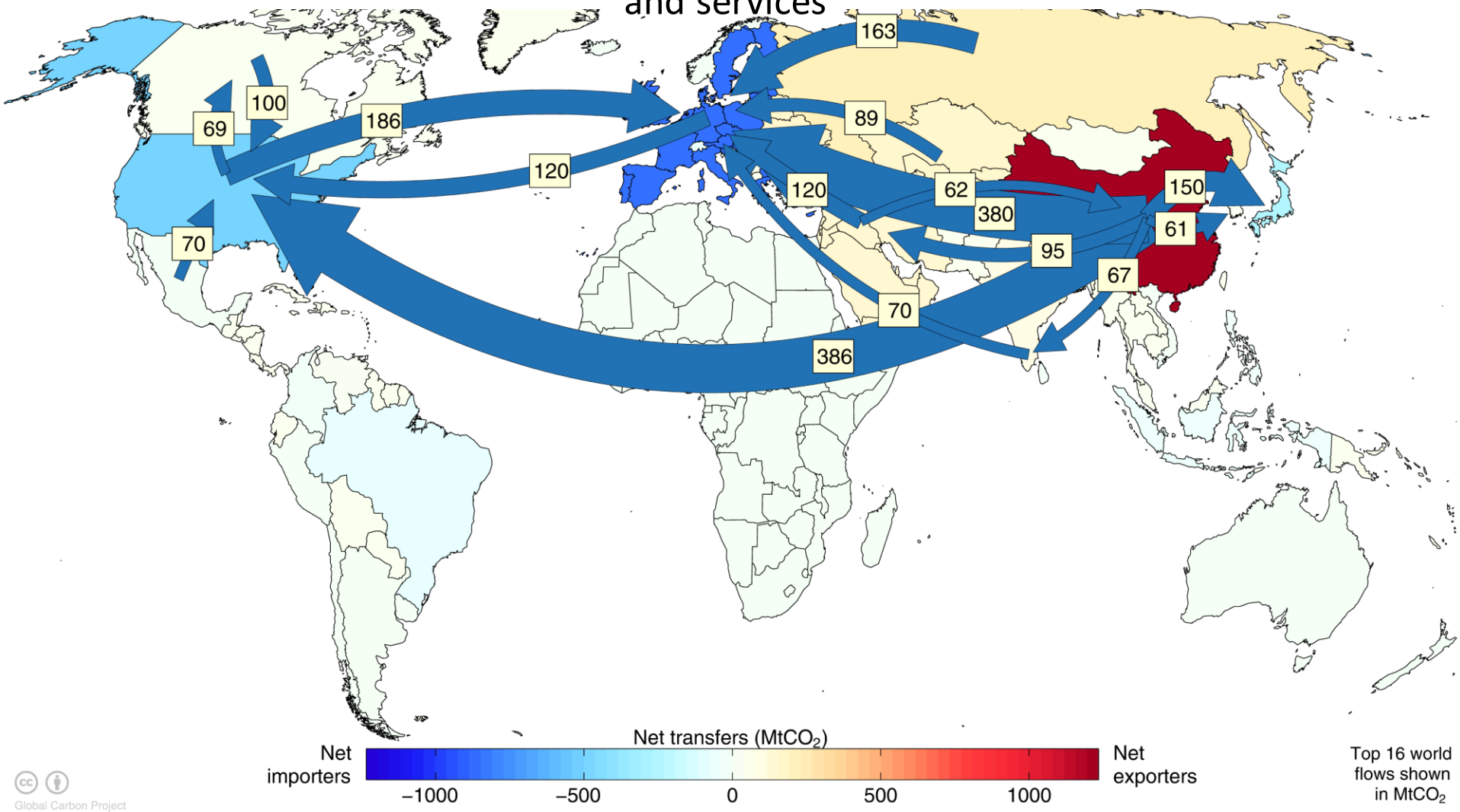
Allocating fossil and industry emissions to the consumption of products provides an alternative perspective.

USA and EU28 are net importers of embodied emissions, China and India are net exporters.



Consumption-based emissions are calculated by adjusting the standard production-based emissions to account for international trade

Flows from location of generation of emissions to location of consumption of goods and services



2 minute discussion

How can we assign emissions responsibility to consumers?

What do we need to measure?

What volume of GHGs are emitted by which industries?

Countries report their emissions by industry to the UNFCCC

How are products made?
Which industries are involved?

Need to understand production recipes and global trade

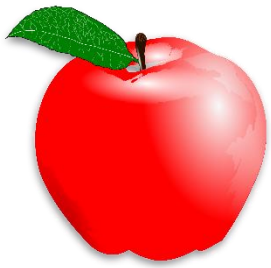
?

How are products bought?
Who buys them?
From where?

Spend by households and governments by product is used to calculate GDP

What are the GHG emissions associated with spending £38 on fruit and veg?

Stage 0



Fruit and veg industry uses 5 tonnes GHG to produce £92 of fruit and veg output.
e_fruit_&_veg = 0.05 tGHG/£.
e is a measure of emissions intensity

Spend on fruit and veg (y_fruit & veg) is £38
Impact is 38 x 0.05 = 2.06 tGHG.
y is a measure of final demand

Note – we find the impact of £1 and multiply by the spend

BUT! What about the rest of the supply chain?? There are other stages involved in the production of a fruit and veg product! The impact is more than 2.06 tGHG

School of Earth & Environment

5582: Tools and Techniques in Ecological Economics

UNIVERSITY OF LEEDS

This is an input output table!

Production recipe

Who else buys products?

Where are items sold to?

What other costs are there for industry?

	Fruit and Veg	Fertiliser	Tractor	Wheels	Rubber	Hhold & Govt spend	Total
Fruit and Veg	50	1	1	1	1	38	92
Fertiliser	10	20	1	1	1	11	54
Tractor	10	1	25	1	1	25	63
Wheels	1	1	20	15	1	10	48
Rubber	1	1	1	20	30	1	54
Taxes & wages	20	30	15	10	20		
Total	92	44	63	48	54		
GHGs	5	10	12	7	8		

Here is the £38 spent on fruit and veg

Here is the 5 tonnes of GHGs emitted by the fruit and veg industry

First lets use the production recipe to find out what else is needed to make £1 of fruit veg.
The table of proportions is **A**

	Fruit and Veg	Fertiliser	Tractor	Wheels	Rubber
Fruit and Veg	0.54	0.02	0.02	0.02	0.02
Fertiliser	0.11	0.45	0.02	0.02	0.02
Tractor	0.11	0.02	0.40	0.02	0.02
Wheels	0.01	0.02	0.32	0.31	0.02
Rubber	0.01	0.02	0.02	0.42	0.56
Taxes & wages	0.22	0.45	0.24	0.21	0.37
Total	1	1	1	1	1
GHGs	0.05	0.23	0.19	0.15	0.15

Stage 1 of the supply chain to make £1 of fruit and veg involves:

0.54 x 0.05 (fruit & veg)
+
0.11 x 0.23 (fertiliser)
+
0.11 x 0.19 (tractor)
+
0.01 x 0.15 (wheels)
+
0.01 x 0.15 (rubber)

= 0.078 tGHGs

So £38 is 2.97 tGHGs

Now we have £38 of spend on fruit and veg producing:

2.06 tGHG in Stage 0 (on site)

2.97 tGHG in Stage 1 (emissions from intermediate industries serving the fruit and veg industry)

BUT! The tractor bought by the fruit&veg industry also has a supply chain!

And the fertiliser has to be made…

And the tractor has to have its wheels made…

	Fruit and Veg	Fertiliser	Tractor	Wheels	Rubber
Fruit and Veg	0.54	0.02	0.02	0.02	0.02
Fertiliser	0.11	0.45	0.02	0.02	0.02
Tractor	0.11	0.02	0.40	0.02	0.02
Wheels	0.01	0.02	0.32	0.31	0.02
Rubber	0.01	0.02	0.02	0.42	0.56
Taxes & wages	0.22	0.45	0.24	0.21	0.37
Total	1	1	1	1	1
GHGs	0.05	0.23	0.19	0.15	0.15

Stage 2 of the supply chain to make £0.11 of tractor involves:

0.11 x 0.02 x 0.05

+

0.11 x 0.02 x 0.23

+

0.11 x 0.40 x 0.19

+

0.11 x 0.32 x 0.15

+

0.11 x 0.02 x 0.15

= 0.014 tGHGs

So £38 is 0.54 tGHGs

Now we have £38 of spend on fruit and veg producing:

2.06 tGHG in Stage 0 (on site)

2.97 tGHG in Stage 1 (emissions from intermediate industries serving the fruit and veg industry)

0.54 tGHG in Stage 2 (emissions from making the tractor used by the fruit and veg industry)

The full supply chain emissions (tracing all paths) for £38 of fruit and veg is actually

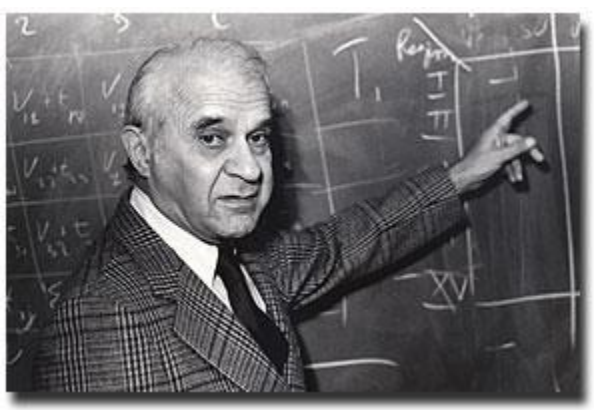
15.48 tGHG!

What does this look like in algebra?

$$Q_1 = e_1 . y_1 + e_1 . A_{11} . y_1 + e_2 . A_{21} . y_1 + e_3 . A_{31} . y_1 + e_4 . A_{41} . y_1 + e_5 . A_{41} . y_1 + e_1 . A_{11} . A_{11} . y_1 + e_2 . A_{21} . A_{11} . y_1 + e_3 . A_{31} . A_{11} . y_1 + \text{etc}$$

More simply

$$\mathbf{Q = eIy + eAy + eA^2y + eA^3y + \ \ldots + eA^ny}$$
$$\mathbf{Q = e\left(I + A + A^2 + A^3 + \cdots + A^n\right)y}$$
$$\mathbf{Q = e(I - A)^{-1}y}$$
$$\mathbf{Q = eLy}$$



This is the environmentally-extended Leontief inverse equation!

What volume of GHGs are emitted by which industries?

How are products made?
Which industries are involved?

How are products bought?
Who buys them?
From where?

Row vector of emissions per £ of industrial output

x

Leontief Inverse – a matrix which reallocates money on industrial production to money on consumption of products

x

Final demand vector showing spend by final consumers

=

Consumption emissions

e **L** **y** **Q**

But our fruit and veg example did not have any importing countries in it…

Country A's Domestic transactions	Country A's industry used to make Country B's products	Country A's industry used to make Country C's products	A's FD from A	B's FD from A	C's FD from A	Total Output
Country B's industry use make Countr products	Country B's	Country B's			C's FD from B	
Country C's industry used to make Country A's products	Country C's industry used to make Country B's products	Country C's Domestic transactions	A's FD from C	B's FD from C	C's FD from C	
VA from A	VA from B	VA from C				
Total output						

A multi-regional input output database

2 minute discussion

Can you think of any issues there might be in constructing a table using data from every country in the world?

- Issues
- Sectoral classifications & meaning
- Currencies
- Bilateral trade mismatches
- Missing data

School of Earth & Environment

5582: Tools and Techniques in Ecological Economics

UNIVERSITY OF LEEDS

School of Earth & Environment

5582: Tools and Techniques in Ecological Economics

UNIVERSITY OF LEEDS

- Consider a world with two countries: Westeros and Essos
- Each country has 3 industries: food, machines, energy
- Let's construct a multi-regional input-output database!

School of Earth & Environment

5582: Tools and Techniques in Ecological Economics

UNIVERSITY OF LEEDS

	Westeros Food	Westeros Machines	Westeros Energy	Essos Food	Essos Machines	Essos Energy		Westeros Demand	Essos Demand		Total
Westeros Food	660	10	2	30	1	1		690	60		1454
Westeros Machines	300	500	100	20	30	20		380	20		1370
Westeros Energy	200	400	300	10	30	100		330	15		1385
Essos Food	40	2	1	440	20	2		200	580		1285
Essos Machines	20	200	200	140	400	90		90	200		1340
Essos Energy	10	30	200	200	200	200		100	280		1220
Value added	224	228	582	445	659	807					
Total	1454	1370	1385	1285	1340	1220					
GHG	980	1000	1200	1000	1100	1550					

What are the production-based emissions for Westeros?

What are the production-based emissions for Essos?

What happens when we find the consumption-based emissions for each nation?

	Westeros Food	Westeros Machines	Westeros Energy	Essos Food	Essos Machines	Essos Energy
Westeros Food	0.45	0.01	0.00	0.02	0.00	0.00
Westeros Machines	0.21	0.36	0.07	0.02	0.02	0.02
Westeros Energy	0.14	0.29	0.22	0.01	0.02	0.08
Essos Food	0.03	0.00	0.00	0.34	0.01	0.00
Essos Machines	0.01	0.15	0.14	0.11	0.30	0.07
Essos Energy	0.01	0.02	0.14	0.16	0.15	0.16
Value added	0.15	0.17	0.42	0.35	0.49	0.66
Total	1	1	1	1	1	1
e	0.67	0.73	0.87	0.78	0.82	1.27

Westeros Demand	Essos Demand
690	60
380	20
330	15
200	580
90	200
100	280

This is $\mathbf{A}$ but we need to calculate $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$

Now we have $\mathbf{y_w}$ and $\mathbf{y_e}$ and we will use each vector to fine the CBA of each nation

We have calculated the emissions intensity $\mathbf{e}$, here. Note how different the intensities are

$$\mathbf{I} - \mathbf{A} = (\mathbf{I} - \mathbf{A})$$

$\mathbf{I}$

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

-

$\mathbf{A}$

0.45	0.01	0.00	0.02	0.00	0.00
0.21	0.36	0.07	0.02	0.02	0.02
0.14	0.29	0.22	0.01	0.02	0.08
0.03	0.00	0.00	0.34	0.01	0.00
0.01	0.15	0.14	0.11	0.30	0.07
0.01	0.02	0.14	0.16	0.15	0.16

=

$(\mathbf{I} - \mathbf{A})$

0.55	-0.01	0.00	-0.02	0.00	0.00
-0.21	0.64	-0.07	-0.02	-0.02	-0.02
-0.14	-0.29	0.78	-0.01	-0.02	-0.08
-0.03	0.00	0.00	0.66	-0.01	0.00
-0.01	-0.15	-0.14	-0.11	0.70	-0.07
-0.01	-0.02	-0.14	-0.16	-0.15	0.84

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{L}$$

$(\mathbf{I} - \mathbf{A})^{-1}$

$$\left(\begin{matrix} 0.55 & -0.01 & 0.00 & -0.02 & 0.00 & 0.00 \\ -0.21 & 0.64 & -0.07 & -0.02 & -0.02 & -0.02 \\ -0.14 & -0.29 & 0.78 & -0.01 & -0.02 & -0.08 \\ -0.03 & 0.00 & 0.00 & 0.66 & -0.01 & 0.00 \\ -0.01 & -0.15 & -0.14 & -0.11 & 0.70 & -0.07 \\ -0.01 & -0.02 & -0.14 & -0.16 & -0.15 & 0.84 \end{matrix}\right)^{-1} =$$

$\mathbf{L}$

1.85	0.03	0.01	0.07	0.01	0.00
0.69	1.69	0.18	0.09	0.07	0.06
0.61	0.67	1.39	0.11	0.10	0.16
0.09	0.02	0.01	1.53	0.04	0.01
0.34	0.52	0.36	0.32	1.50	0.18
0.22	0.26	0.31	0.36	0.29	1.26

So now we have all the component parts we need!

Row vector of emissions per £ of industrial output	x	Leontief Inverse – a matrix which reallocates money on industrial production to money on consumption of products	x	Final demand vector showing spend by final consumers	=	Full supply chain emissions
$\mathbf{e}$		$\mathbf{L}$		$\mathbf{y}$		$\mathbf{Q}$

$$\mathbf{Q}_{\mathbf{w}} = \mathbf{eLy}_{\mathbf{w}}$$

$$\mathbf{Q}_{\mathbf{e}} = \mathbf{eLy}_{\mathbf{e}}$$

But… matrices don't quite work so easily

Matrix of emissions per £ of industrial output

(put emissions intensity on the diagonal)

$\hat{e}$

x

Leontief Inverse – a matrix which reallocates money on industrial production to money on consumption of products

L

x

Final demand matrix showing spend by final consumers

(put each final demand vector on a diagonal)

$\hat{y}$

=

Full supply chain emissions by end product and source industry

Q

Column sum gives product impacts

Row sum gives source impacts

- Exercises 1-3 (20 mins)
  - Create the data needed for the Game of Thrones input-output table
  - Make A the total requirements matrix
  - Make L the Leontief inverse

http://www.see.leeds.ac.uk/teaching-resources/

- Exercise 4 to 5 (40 mins)
  - Generate consumption-based emissions for Westeros and Essos

$$\mathbf{Q_w} = \mathbf{\hat{e}L\widehat{y_w}}$$

Emissions in Westeros to make Westeros products bought by Westeros	Emissions in Westeros to make Essos products bought by Westeros
Emissions in Essos to make Westeros products bought by Westeros	Emissions in Essos to make Essos products bought by Westeros

$$\mathbf{Q_e} = \mathbf{\hat{e}L\widehat{y_e}}$$

Emissions in Westeros to make Westeros products bought by Essos	Emissions in Westeros to make Essos products bought by Essos
Emissions in Essos to make Westeros products bought by Essos	Emissions in Essos to make Essos products bought by Essos

- Exercise 7(30 mins)
  - Generate graphs of Westeros and Essos' consumption-based accounts

$$\mathbf{Q_w} = \mathbf{\hat{e}L\widehat{y_w}}$$

Emissions in Westeros to make Westeros products bought by Westeros	Emissions in Westeros to make Essos products bought by Westeros
Emissions in Essos to make Westeros products bought by Westeros	Emissions in Essos to make Essos products bought by Westeros

$$\mathbf{Q_e} = \mathbf{\hat{e}L\widehat{y_e}}$$

Emissions in Westeros to make Westeros products bought by Essos	Emissions in Westeros to make Essos products bought by Essos
Emissions in Essos to make Westeros products bought by Essos	Emissions in Essos to make Essos products bought by Essos

Thank you

Dr Anne Owen
a.owen@leeds.ac.uk
@dr_anneowen